AGS 3 Name: ______________________ Period: _____ Date: __________

Notes 3.7 – Polynomial Roots

Warmup – Polynomial vs Not a Polynomial – Why?

Compare each set of functions, then explain how they are different. You can use either a graphing calculator or desmos.com to help you.

1. a. $f(x) = x^3$

cubic

3 roots

domain & range both $\mathbb{R}$

b. $g(x) = 3^x$

exponential

no roots

domain $\mathbb{R}$

range $(0, \infty)$

2. a. $f(x) = 2x^2 + 5x - 12$

quadratic

2 roots

one continuous graph

b. $g(x) = \frac{2x^2}{x^2-3x+2}$

rational

1 root

3 parts to graph

3. a. $f(x) = \frac{1}{2}x$

linear

1 root

b. $g(x) = \frac{1}{2x}$

rational

no roots

2 parts to graph

4. a. $f(x) = x^2$

quadratic

domain $\mathbb{R}$

b. $g(x) = x^{\frac{1}{2}}$

square root

limited domain

Investigation

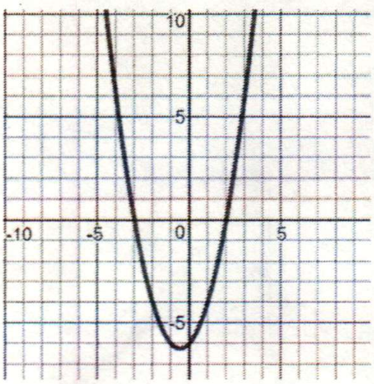
Quadratics are a type of polynomial. Find the roots (x-intercepts) for each equation and identify if they are real or imaginary.

a. $f(x) = x^2 + x - 6$

$(x+3)(x-2)$

$\boxed{x = -3, 2}$

$\boxed{\text{both real}}$

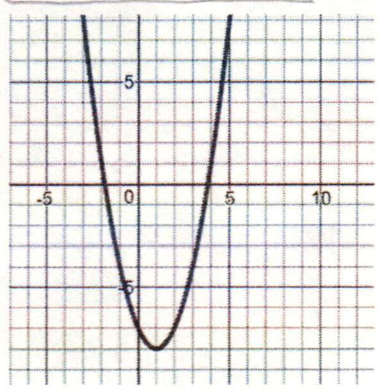


b. $g(x) = x^2 - 2x - 7$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{32}}{2}$ $\boxed{x = 1 \pm 2\sqrt{2}}$

$\boxed{\text{both real}}$



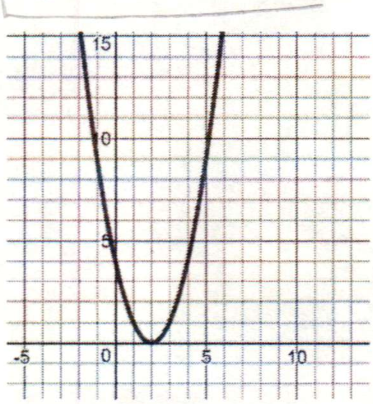
c. $h(x) = x^2 - 4x + 4$

$(x-2)^2$

$\searrow (x-2)(x-2)$

$\boxed{x = 2}$

$\boxed{\text{both real}}$

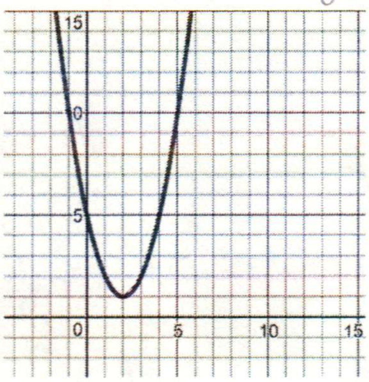


d. $k(x) = x^2 - 4x + 5$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{-4}}{2}$ $\boxed{x = 2 \pm 1i}$

2 imaginary



Write the equations of each of the quadratics in factored form.

$f(x) =$ $(x+3)(x-2)$

$g(x) =$ $(x-(1+2\sqrt{2}))(x-(1-2\sqrt{2}))$

$h(x) =$ $(x-2)^2$

$(x-2)(x-2)$

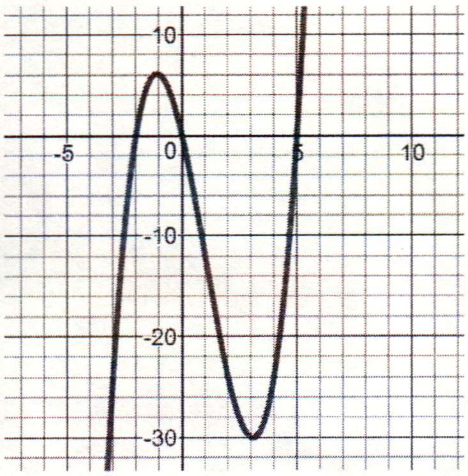
$k(x) =$ $(x-(2+i))(x-(2-i))$

What is the relationship between roots and factors?

Roots can be used to write the factors

A cubic function is also a polynomial. Compare the equation of this cubic function to its graph.

$m(x) = x^3 - 3x^2 - 10x$



e. Identify the roots of the function. Show how you can verify the roots are correct.

$x = -2, 0, 5$

$y = (-2)^3 - 3(-2)^2 - 10(-2) = 0$

$y = (0)^3 - 3(0)^2 - 10(0) = 0$

$y = (5)^3 - 3(5)^2 - 10(5) = 0$

f. Write the equation in factored form.

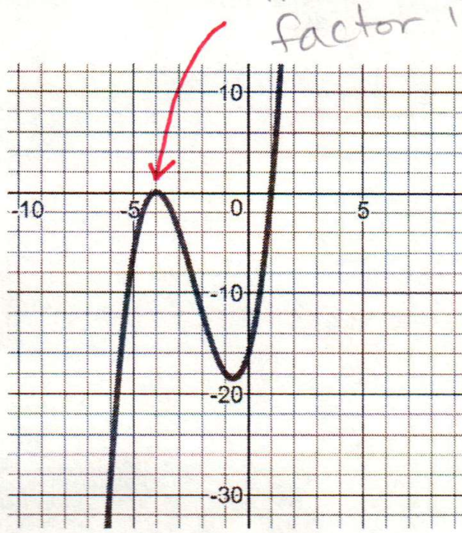
$m(x) =$ $(x-(-2))(x-0)(x-5)$

$y = (x+2)(x)(x-5)$

Now compare this cubic function with its graph.

a bounce means that factor is repeated

$n(x) = x^3 + 7x^2 + 8x - 16$



g. Identify the roots.

$x = -4, 1$

h. Write the equation in factored form.

$n(x) = (x+4)^2(x-1)$

i. Verify your equation in factored form is correct by converting it into standard form.

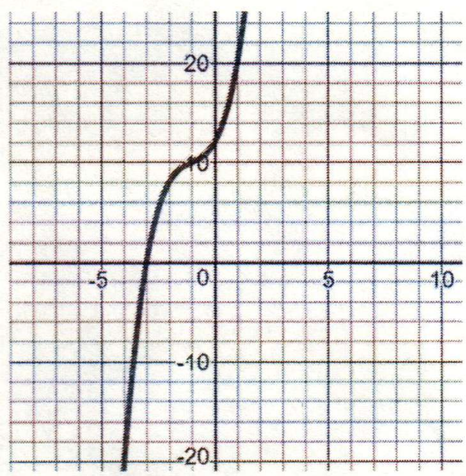
$(x^2+8x+16)(x-1)$

$\boxed{n(x) = x^3 + 7x^2 + 8x - 16}$

	x^2	$8x$	16
x	x^3	$8x^2$	$16x$
-1	$-x^2$	$-8x$	-16

This cubic function has less obvious roots.

$p(x) = (x+3)(x^2+4)$



j. Find the roots of the function.

$x + 3 = 0$ $\quad x^2 + 4 = 0$

$x = -3$ $\quad x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

Vocabulary

Word	Meaning/Notation	Example
Fundamental Theorem of Algebra	A polynomial of n^{th} degree has n roots	x^5 has 5 roots
Roots	another name for x-intercepts	$(x, 0)$
Factored Form	$y = (x-p)(x-q)(x-r)$, etc roots $= p, q, r$	